

J11 BA pps 30-43

Answers

P30

[1.1]  $220^\circ$

[1.2]  $-45$

[1.3]  $240^\circ$

P31

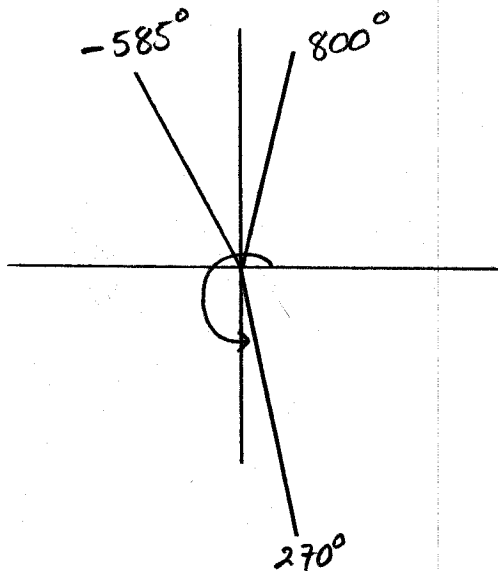
[2.1] 8 PM - 10 AM is 14 HRS

minute hand  $14 \cdot 360^\circ = 5040^\circ$

Hour hand  $360^\circ + \frac{2}{12}(360) = 420^\circ$

[3]  $-\beta$

[4]



P32

[5.1]  $30^\circ + 360n, n \in \mathbb{Z}$

[5.2]  $180^\circ + 360n, n \in \mathbb{Z}$

[5.3]  $315 + 360n, n \in \mathbb{Z}$

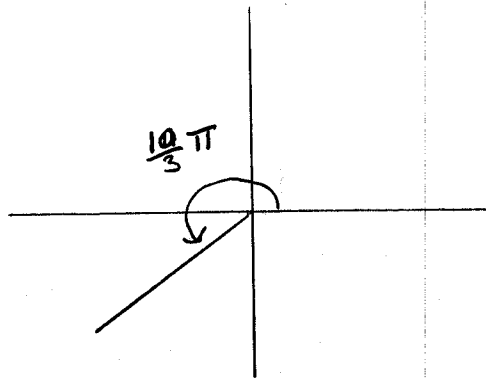
P.33

[6]  $30^\circ = \frac{\pi}{6}, 45^\circ = \frac{\pi}{4}, 150^\circ = \frac{5\pi}{6}, 360^\circ = 2\pi$

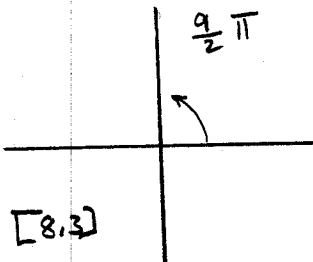
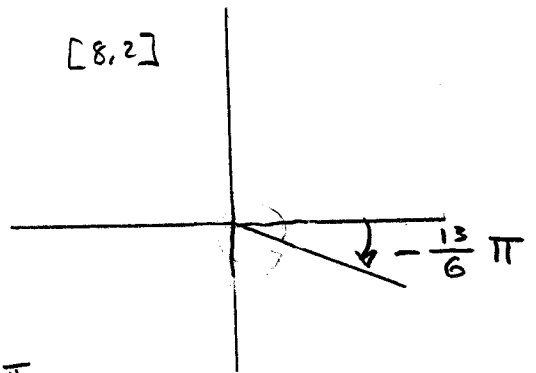
[7]  $\frac{\pi}{5} = 36^\circ, \frac{2}{3}\pi = 120^\circ, \frac{3}{2}\pi = 270^\circ, 3\pi = 540^\circ$

P 33, ctd

[8.1]



[8.2]



[8.3]

[9.1]  $\frac{\pi}{3} + 2n\pi, n \in \mathbb{Z}$

[9.2]  $-\frac{3\pi}{4} + 2n\pi, n \in \mathbb{Z}$

[9.3]  $3\pi + 2n\pi, n \in \mathbb{Z}$  OR

Pps 34.

$$\#11 \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

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$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

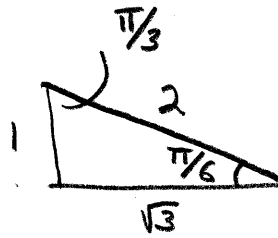
$$\tan \frac{\pi}{4} = 1$$


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$$\sin \frac{\pi}{6} = \frac{1}{2}$$

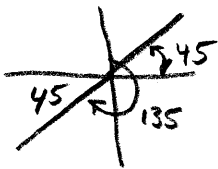
$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$



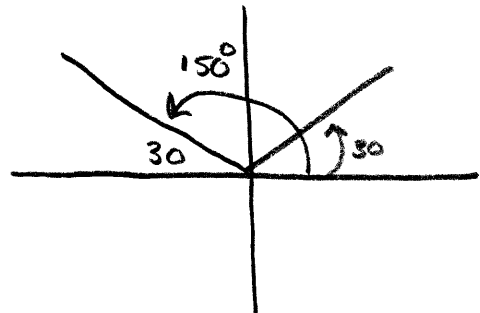
$$\#5 \quad 1) \cos(-60) = \cos 60 = \frac{1}{2}$$

$$2) \sin(-135) = -\sin 45 = -\frac{\sqrt{2}}{2}$$

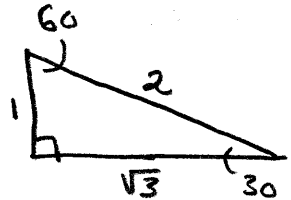
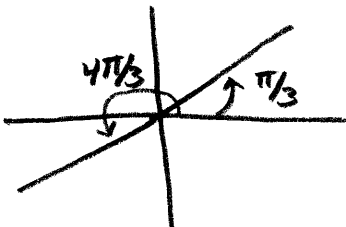


$$3) \tan 870 = \tan 150$$

$$= -\tan 30 = -\frac{\sqrt{3}}{3}$$

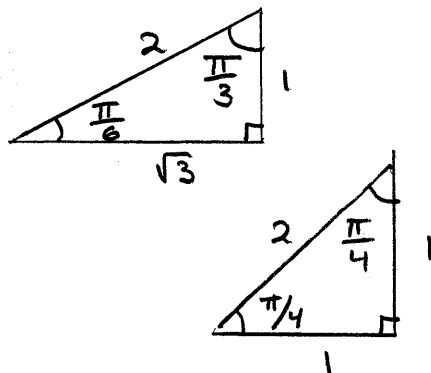


$$4) \sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$



(Prob 1)

	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
sin	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
cos	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
tan	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$



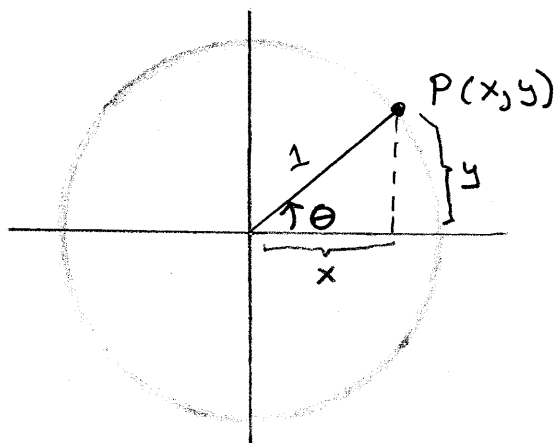
(Prob 2)

$$\text{I. } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

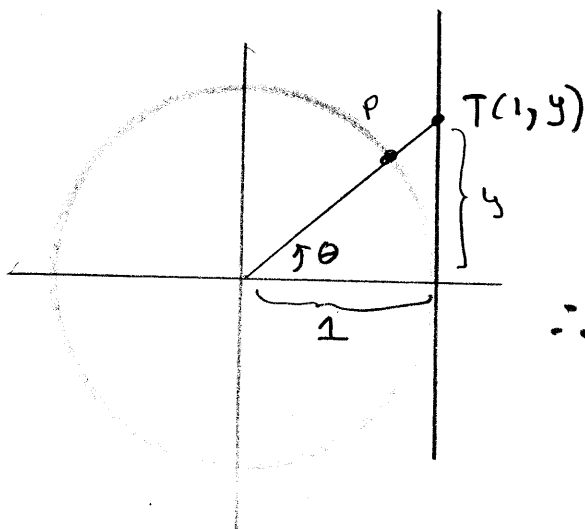
$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin \theta}{\cos \theta} \quad \checkmark$$

(Prob 3)

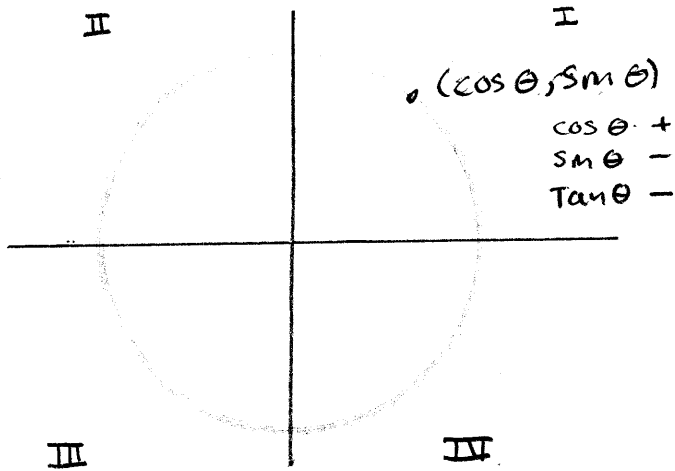


$$\begin{aligned} \cos \theta &= \frac{x}{1}, \quad \sin \theta = \frac{y}{1} \\ \Rightarrow x &= \cos \theta, \quad y = \sin \theta \quad \checkmark \\ \therefore P &= (\cos \theta, \sin \theta) \end{aligned}$$



$$\begin{aligned} \tan \theta &= \frac{y}{1} \\ y &= \tan \theta \\ \therefore T &= (1, \tan \theta) \end{aligned}$$

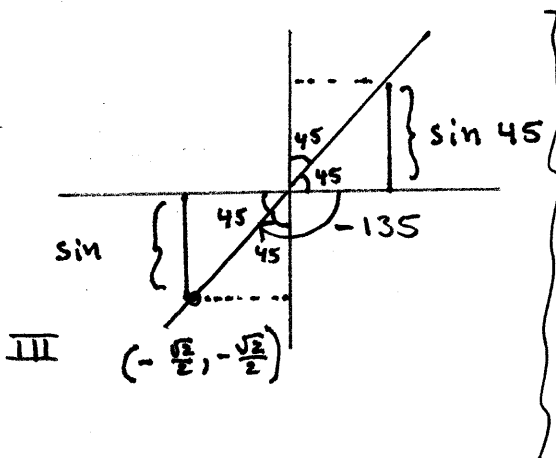
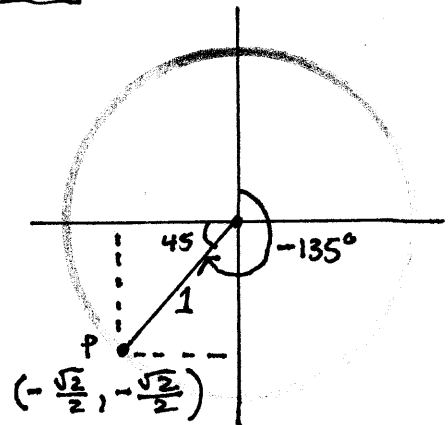
(Prob 4)



(Prob 5) KEY TO ALL is to find coords. of P.

$$[1] \cos(-60^\circ) = \underbrace{\cos(-\pi/3)}_{\text{angle in IV Quad}} = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$[2] \sin(-135^\circ) = -\sin 135^\circ$$

**Better**

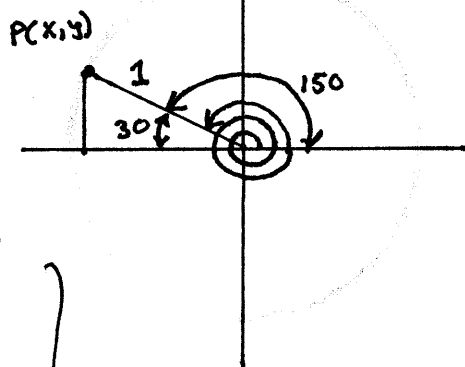
$$\begin{aligned} \text{Coords of P } & (\cos -135^\circ, \sin -135^\circ) \\ & = P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \end{aligned}$$

$$\therefore \sin(-135^\circ) = \frac{-\sqrt{2}}{2}$$

Pg 36 ctd

(Prob 5) ctd

[3]  $\tan 870^\circ$



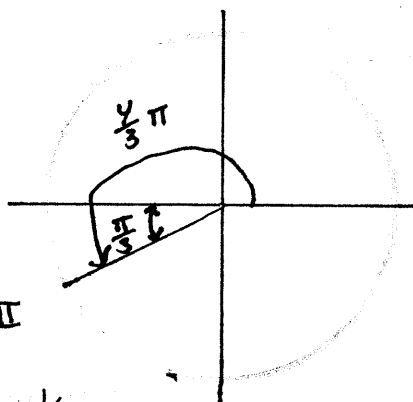
$$870^\circ = 2(360^\circ) + 150^\circ$$

y coord is pos and the sin of  $30^\circ$ , so  $y = \sin 30 = \frac{1}{2}$

x coord is NEG and is  $\cos 30 = \frac{\sqrt{3}}{2}$

$$\therefore \tan 870 = \tan 150 = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

[4]  $\sin \frac{4}{3}\pi$



coords

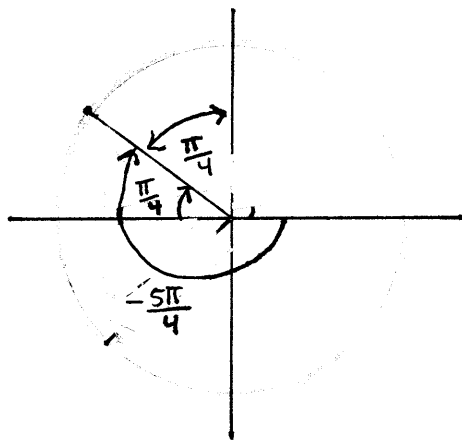
$$x = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$y = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\therefore \sin \frac{4}{3}\pi = -\frac{\sqrt{3}}{2}$$

[5]  $\tan(-\frac{5\pi}{4})$

$$\frac{5}{4}\pi = \pi + \frac{1}{4}\pi$$



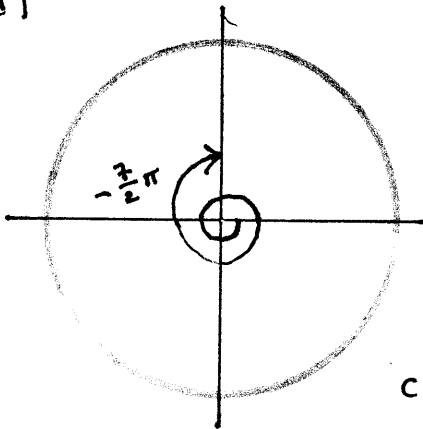
$$\cos(-\frac{5\pi}{4}) = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin(-\frac{5\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\therefore \tan(-\frac{5\pi}{4}) = -1$$

(Prob 5 ctd)

[6]



$$\begin{aligned} -\frac{7}{2}\pi &= -\frac{6}{2}\pi - \frac{1}{2}\pi \\ &= -3\pi - \frac{1}{2}\pi \end{aligned}$$

$$\cos\left(-\frac{7}{2}\pi\right) = \cos\frac{\pi}{2} = 0$$

Pg 37

(Prob 6)

[1] Prove  $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$

proof:  $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\therefore \tan^2 \theta + 1 = \frac{1}{\cos^2 \theta}$$

(2) Prove  $\frac{1 - 2\sin \theta \cos \theta}{1 + 2\sin \theta \cos \theta} = \left[ \frac{1 - \tan \theta}{1 + \tan \theta} \right]^2$

Proof: RHS (TOP) =  $(1 - \tan \theta)^2 = 1 - 2\tan \theta + \tan^2 \theta = 1 - 2\frac{\sin \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$   
 $= \frac{\cos^2 \theta - 2\sin \theta \cos \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1 - 2\sin \theta \cos \theta}{\cos^2 \theta}$

RHS (BOT) =  $\frac{1 + 2\sin \theta \cos \theta}{\cos^2 \theta}$

$$\text{RHS} = \frac{\frac{1 - 2\sin \theta \cos \theta}{\cos^2 \theta}}{\frac{1 + 2\sin \theta \cos \theta}{\cos^2 \theta}} = \frac{1 - 2\sin \theta \cos \theta}{1 + 2\sin \theta \cos \theta} = \text{LHS} \quad \square$$

Pg 37 ctd

(Prob 6) ctd

[3] Prove:  $\sin^4 \theta - \cos^4 \theta = 2 \sin^2 \theta - 1$

$$\text{LHS} = \sin^4 \theta - \cos^4 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$$

$$= \sin^2 \theta - \cos^2 \theta$$

$$= \sin^2 \theta - \cos^2 \theta + \sin^2 \theta - \sin^2 \theta$$

$$= 2 \sin^2 \theta - (\cos^2 \theta + \sin^2 \theta)$$

$$\therefore = 2 \sin^2 \theta - 1$$

□

(Prob 7)

$\sin \theta = -\frac{3}{5}$ ,  $\theta$  in Quad IV

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta + \left[-\frac{3}{5}\right]^2 = 1$$

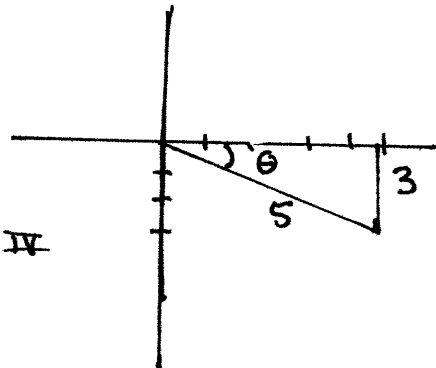
$$\cos^2 \theta = 1 - \frac{9}{25}$$

$$= \frac{16}{25}$$

$\cos \theta = \frac{4}{5}$  and  $\cos \theta$  pos for  $\theta$  in IV

$$\tan \theta = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$$

$\tan \theta = -\frac{3}{4}$



IV

(Prob 8)

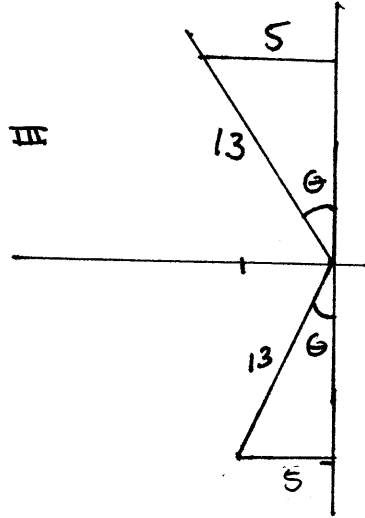
$$\sqrt{\cos \theta = -\frac{5}{13}}$$

$$\Rightarrow \theta \text{ in Quad II OR III}$$

$$\sin^2 \theta + \left(-\frac{5}{13}\right)^2 = 1$$

$$\sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\sin \theta = \frac{12}{13}$$



Quad II

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = -\frac{5}{13}$$

$$\tan \theta = -\frac{12}{5}$$

QUAD III

$$\sin \theta = -\frac{12}{13}$$

$$\cos \theta = -\frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

.j 38

(Prob 1)

$$a) \sin 6\pi = \sin(3 \cdot 2\pi) = \sin 0 = 0$$

$$b) \sin \frac{11}{3}\pi = \sin\left(4\pi - \frac{\pi}{3}\right)$$

$$= \sin\left(-\frac{\pi}{3}\right) = -\sin \frac{\pi}{3} = \left(-\frac{\sqrt{3}}{2}\right)$$

$$\frac{11}{3} = 3\frac{2}{3} = 2\frac{5}{3}$$

$$2\pi = \frac{6\pi}{3}$$

$$4\pi = \frac{12}{3}\pi$$

$$c) \cos\left(-\frac{23}{6}\pi\right) = \cos\left(-\frac{\pi}{6} + 4\pi\right) = \cos\left(-\frac{\pi}{6}\right) = \cos \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)$$

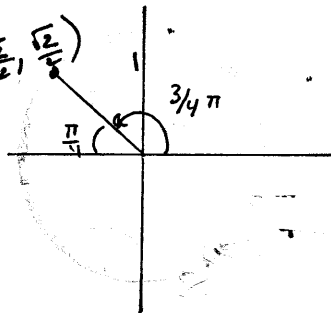
$$d) \tan\left(-\frac{27}{4}\pi\right) = \tan\left(-\frac{3}{4}\pi - 6\pi\right)$$

$$= \tan\left(-\frac{3}{4}\pi\right) \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$= -\tan\left(\frac{3}{4}\pi\right)$$

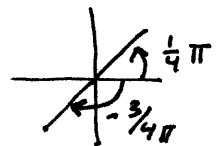
$$= -(-1)$$

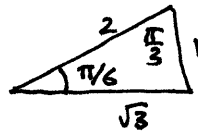
$$= 1$$



$$\frac{28}{4} = 7$$

$$24 \quad 3 \quad 32$$

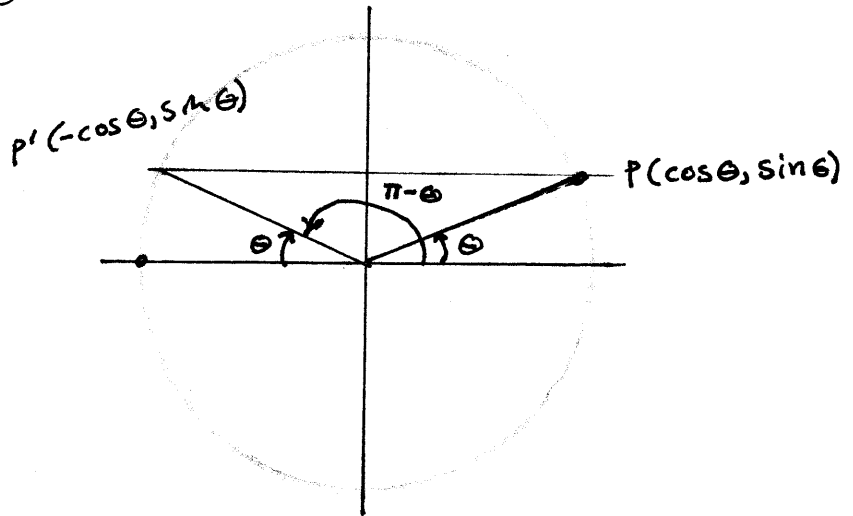




(Prob 2)

Prove ①  $\sin(\pi - \theta) = \sin \theta$ ②  $\cos(\pi - \theta) = -\cos \theta$ ③  $\tan(\pi - \theta) = -\tan \theta$ 

① P, P' sym wrt y-axis  
so P'  $(-\cos \theta, \sin \theta)$

①  $\therefore \sin(\pi - \theta) = \sin \theta$ ②  $\cos(\pi - \theta) = -\cos \theta$ ③  $\tan(\pi - \theta) = -\tan \theta$ 

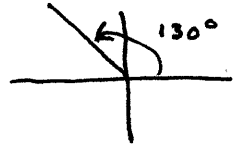
(Prob 3)

$$\left\{ \begin{array}{l} \sin \frac{17\pi}{6} = \sin \left( \frac{18\pi}{6} - \frac{\pi}{6} \right) = \sin \left( 3\pi - \frac{\pi}{6} \right) = \sin \left( \pi - \frac{\pi}{6} \right) = \sin \frac{\pi}{6} = \frac{1}{2} \\ \cos \frac{17\pi}{6} = \cos \left( \frac{18\pi}{6} - \frac{\pi}{6} \right) = \cos \left( 3\pi - \frac{\pi}{6} \right) = \cos \left( \pi - \frac{\pi}{6} \right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2} \\ \tan \frac{17\pi}{6} = \frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \end{array} \right.$$

$$\left\{ \begin{array}{l} \sin \left( -\frac{14\pi}{3} \right) = \sin \left( -\frac{15\pi}{3} + \frac{\pi}{3} \right) = \sin \left( -5\pi + \frac{\pi}{3} \right) = \sin \left( -\pi + \frac{\pi}{3} \right) \\ = \sin - \left( \pi - \frac{\pi}{3} \right) = -\sin \left( \pi - \frac{\pi}{3} \right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2} \\ \cos \left( -\frac{14\pi}{3} \right) = \dots = \cos \left( -\pi + \frac{\pi}{3} \right) = \cos - \left( \pi - \frac{\pi}{3} \right) = \cos \left( \pi - \frac{\pi}{3} \right) \\ = -\cos \frac{\pi}{3} = \boxed{-\frac{1}{2}} \\ \tan \left( -\frac{14\pi}{3} \right) = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3} \end{array} \right.$$

(Prob 3) ctd

$$\begin{cases} \sin\left(\frac{15\pi}{4}\right) = \sin\left(\frac{16\pi}{4} - \frac{\pi}{4}\right) = \sin\left(4\pi - \frac{\pi}{4}\right) = \sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2} \\ \cos\left(\frac{15\pi}{4}\right) = \dots = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \\ \tan\left(\frac{15\pi}{4}\right) = -1 \end{cases}$$



(Prob 4)

$$\begin{aligned} \textcircled{1} \quad \cos 850^\circ &= \cos(360 \cdot 2 + 130^\circ) = \cos(130^\circ) = \cos(180 - 50) = \\ &= \cos 50 = .6428 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin 1315^\circ &= \sin(4 \cdot 360 - 125) = \sin -125 = -\sin 125 \\ &= -\sin(180 - 125) = -\sin 55 = -0.8192 \end{aligned}$$

$$\textcircled{3} \quad \tan(-3430) = \tan(-10 \cdot 360 + 170) = \tan(180 - 10) = -\tan 10 = -.1763$$

(Prob 5)

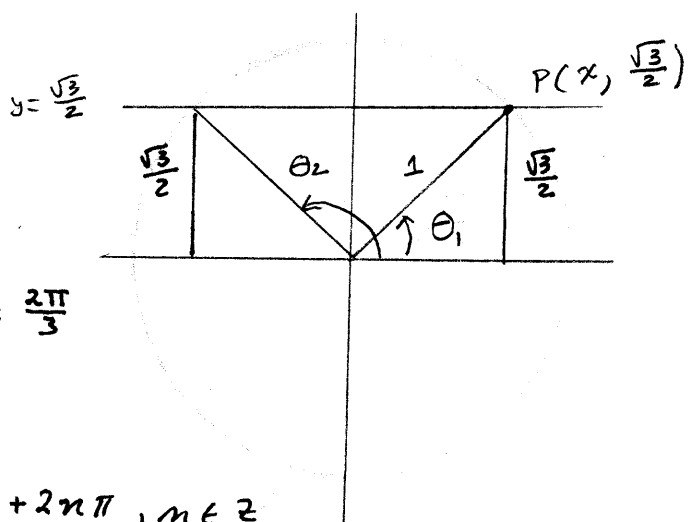
①

$$0 \leq \theta < 2\pi$$

$$\theta = \frac{\pi}{3} \quad \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

So,

$$\theta = \frac{\pi}{3} + 2n\pi \quad \theta = \frac{2\pi}{3} + 2n\pi, n \in \mathbb{Z}$$



pg 43 ctd

(Prob 5) ctd

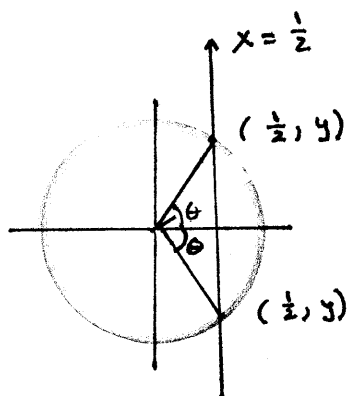
$$\textcircled{2} \cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi$$

$$\theta = -\frac{\pi}{3} + 2n\pi$$



P 43, ctd

[5.3] Solve  $\sin 2\theta = -\frac{1}{\sqrt{2}}$  for  $\theta$

Soln

A  $\sin 2\theta = \frac{-\sqrt{2}}{2}$

P  $\sin\left(\frac{5\pi}{4} + 2n\pi\right) = \frac{-\sqrt{2}}{2}$

Q  $\sin\left(\frac{7\pi}{4} + 2n\pi\right) = \frac{-\sqrt{2}}{2}$

(A, P)  $\Rightarrow \sin 2\theta = \sin\left(\frac{5\pi}{4} + 2n\pi\right)$

$$2\theta = \frac{5\pi}{4} + 2n\pi$$

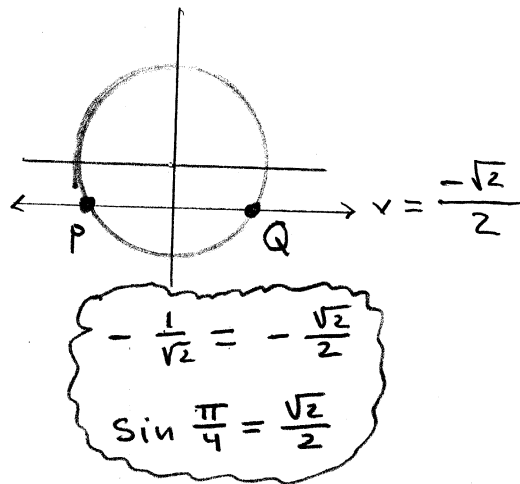
$$\theta = \frac{5\pi}{8} + n\pi$$

(A, Q)  $\Rightarrow \sin 2\theta = \sin\left(\frac{7\pi}{4} + 2n\pi\right)$

$$2\theta = \frac{7\pi}{4} + 2n\pi$$

$$\theta = \frac{7\pi}{8} + n\pi$$

$\therefore \theta \in \left\{x : x = \frac{5\pi}{8} + n\pi, n \in \mathbb{Z}\right\} \cup \left\{x : x = \frac{7\pi}{8} + n\pi, n \in \mathbb{Z}\right\}$



P43, ctd

[5.4] solve  $2 \cos 2\theta = 1$  for  $\theta$ .

Soln

$$2 \cos 2\theta = \frac{1}{2}$$

A  $\cos 2\theta = \frac{1}{2}$

P  $\cos\left(\frac{\pi}{3} + 2n\pi\right) = \frac{1}{2}$

Q  $\cos\left(\frac{5\pi}{3} + 2n\pi\right) = \frac{1}{2}$

$$(A, P) \Rightarrow \cos 2\theta = \cos\left(\frac{\pi}{3} + 2n\pi\right)$$

$$2\theta = \frac{\pi}{3} + 2n\pi$$

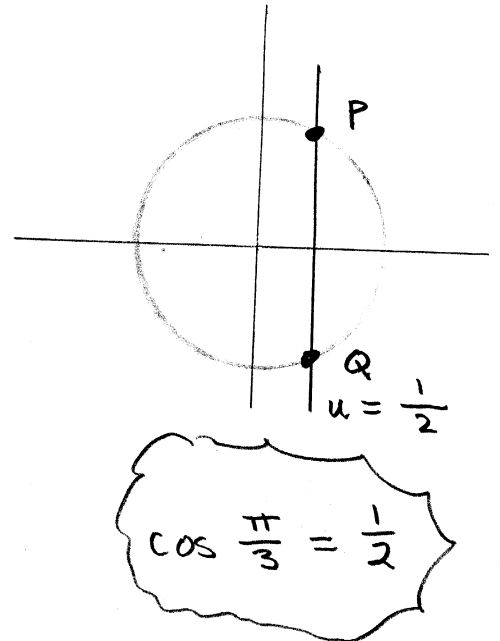
$$\theta = \frac{\pi}{6} + n\pi$$

$$(A, Q) \Rightarrow \cos 2\theta = \cos\left(\frac{5\pi}{3} + 2n\pi\right)$$

$$2\theta = \frac{5\pi}{3} + 2n\pi$$

$$\theta = \frac{5\pi}{6} + n\pi$$

$$\therefore \theta \in \left\{x : x = \frac{\pi}{6} + n\pi, n \in \mathbb{Z}\right\} \cup \left\{x : x = \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}\right\}$$



p 43 ctd

(Prob 6)

$$0 \leq \theta < 2\pi$$

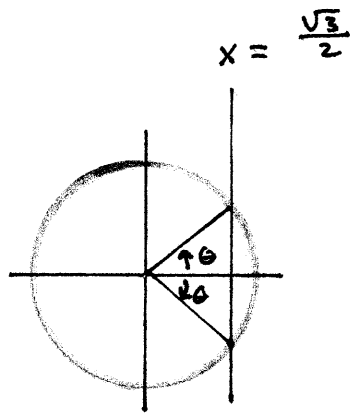
①

$$\cos \theta \leq \frac{\sqrt{3}}{2}$$

$$\theta < \frac{\pi}{6}$$

$$\theta > -\frac{\pi}{6}$$

$$\therefore -\frac{\pi}{6} < \theta < \frac{\pi}{6}$$



②  $\sin \theta > \frac{1}{2}$

$$\theta > \frac{\pi}{6}$$

$$\theta < \pi - \frac{\pi}{6}$$

$$\theta < \frac{5\pi}{6}$$

$$\therefore \frac{\pi}{6} < \theta < \frac{5\pi}{6}$$

